MODULE - 1

INTRODUCTION AND ANALYSIS OF MEMBERS

Pre-stressed concrete

Definition: Concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from given external loadings are countered to a desired degree.

Types of pre-stressing

Pre-tensioning & Post-tensioning

In pre-tensioning the tendons are tensioned before the concrete is placed. The tendons are temporarily anchored to abutments or stressing beds. Then the concrete member is cast between and over the wires. After the concrete has attained the required strength, the wires are cut from the bulkhead and pre-stress is transferred to the concrete member.

In post-tensioning the concrete member is cast with ducts for the wires. After concrete has attained sufficient strength, wires are threaded into the ducts, tensioned from both or one end by means of jack/jacks and at the precise level of pre-stress the wires are anchored by means of wedges to the anchorage plates at the ends.

Bonded & Un-bonded tendon

In post-tensioned members, the wires are either left free to slide in the ducts or the duct is filled with grout. In the former, the tendon is un-bonded and in the latter it is bonded.

The member is under pre-stress but is not subjected to any superimposed external loads. Further subdivision of this stage is possible.

1. Before pre-stressing: Concrete is weak in carrying loads. Yielding of supports must be prevented.

2. During pre-stress:

a. Steel: This stage is critical for the strength of tendons. Often the maximum stress to which the wires will be subjected throughout their life may occur at this stage.

b. Concrete: As concrete has not aged at this stage, crushing of concrete at anchorages is possible, if its quality is inferior or the concrete is honeycombed. Order of pre-stressing is important to avoid overstress in the concrete.

3. At transfer of pre-stress: For pre-tensioned members, where transfer is within a short period, and for post-tensioned members where transfer may be gradual, there are no external loads on the member except its own weight.

4. De-shuttering: The removal of form-work must be done after due consideration

Thus the initial pre-stress with little loss imposes a serious condition n the concrete and often controls the design of the member.

Final stage

This is the stage when actual working loads come on the structure. The designer must consider various combinations of live loads on different parts of the structure with lateral loads such as wind and earthquake forces and strain loads produced by settlement of supports and temperature. The major loads in this stage are:

1. Sustained load: It is often desirable to limit the deflection under sustained loads sue to its own weight and dead loads.
2. Working load: The member must be designed for the working load. Check for excessive stress and deflection must be made. But this design may not guarantee sufficient strength to carry overloads.
3. Cracking load: Cracking in a pre-stress member signifies a sudden change in bond and shearing stresses. This stage is also important
4. Ultimate load: This strength denotes the maximum load the member can carry before collapse.

Analysis of Section

Assumptions:

Prestressed concrete members are analysed and designed on the basis of the assumptio given below

- A transverse plane section of the member will remain a plane after bending also.
- The material concrete is homogenous.
- Within the limits of the deformations taking place, Hook's law is applicable to concrete an steel.
- In case of bonded tendons, the bond between the concrete and steel is perfect that they ac monolithically.
- Upto the level of deformation corresponding to the service loads, concrete is assumed t behave linearly elastic.
- The additional strain/stress in prestressing steel due to flexure caused by external loads i negligible when compared to applied stress due to pre tensioning.

Stress Concept

The combination of the effect of external loads and of prestressing are together as equivalent stresses & compared with the permissible levels of stresses in the material is known as stress concept. This concept is used for checking the design and the permissibility of stresses at various stages of construction during service loads.

(a) Concentric Tendon

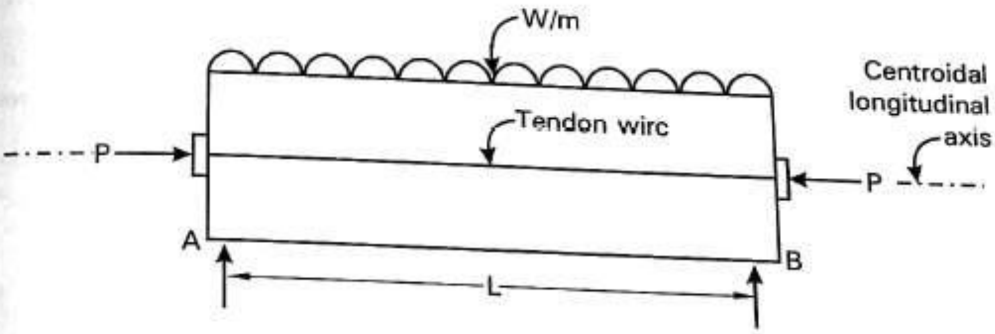


Fig. 1.21 : (a) Prestressed concrete beam with concentric tendon

Fig. 1.21(a) shows a simply supported beam subjected to an external load system. It is prestressed by a tendon provided through its centroidal longitudinal axis.

Let 'P' be the prestressing force supplied by the tendon. Due to prestressing force, the compressive stress induced in concrete = $\frac{P}{A}$

where, A = Area of the c/s of the beam

The stress due to Bending Moment (BM) of this section = $\pm \frac{M}{Z}$

The resultant stress at the top and bottom fibres are

Stress at the extreme top edge (fibre), $f_t = \frac{P}{A} + \frac{M}{Z}$

Stress at the extreme bottom edge (fibre), $f_b = \frac{P}{A} - \frac{M}{Z}$

Fig. 1.21 (b) shows the pressure distribution diagram for this section.

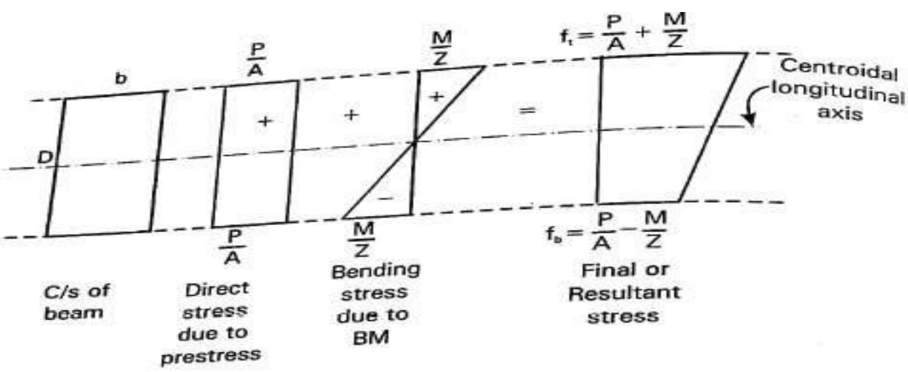


Fig. 1.21(b) : Resultant stress

(b) Eccentric Tendon

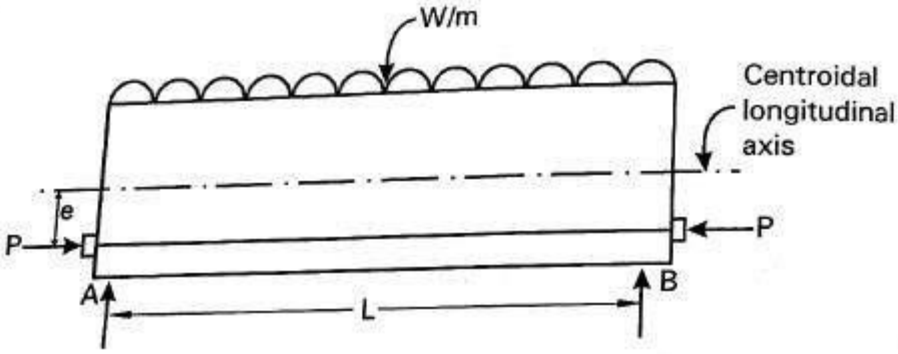
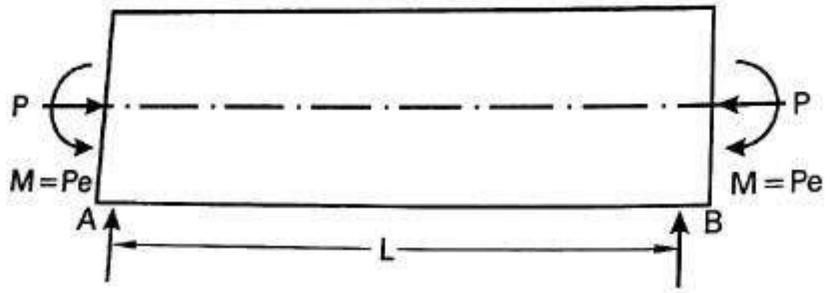


Fig. 1.22 (a)



Consider a simply supported beam as shown in fig. 1.22(a), which is subjected by an external loading system. This beam is prestressed by a tendon placed longitudinaly at an eccentricity 'e' from the centroidal longitudinal axis.

Let P = prestressing force supplied by the tendon.

Let due to the dead load and extreme loads the bending moment at a section be M.

The stresses on the section are as follows ;

1. Direct stress due to prestressing force = $\frac{P}{A}$
2. Bending stresses due to eccentricity of prestressing force = $\frac{Pe}{Z}$
3. Bending stresses due BM = $\frac{M}{Z}$

∴ Resultant stress at top and bottom fibres are

$$f_t = \frac{P}{A} - \frac{Pe}{Z} + \frac{M}{Z} \text{ - stress at the extreme top fibre}$$

$$f_b = \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z} \text{ - stress at the extreme bottom fibre}$$

Where, P = Prestressing force (positive when producing direct compression)

e = Eccentricity of prestressing force

$M = P.e$ = Moment or bending moment

A = cross sectional area of the concrete member

I = second moment of area of section about its centroid

Z = section modulus

f_t = prestress in concrete developed at the top fiber

f_b = prestress in concrete developed at the bottom fiber

Force Concept

If the capacity of the section is decided based on the total tension and compression it carries, is known as Force concept.

This concept is also called strength concept or lever arm concept. In a prestressed concrete beam, the high tensile is tensioned and anchored. As an effect, the tensile stresses and compressive stresses develop in steel and concrete respectively. The internal moment is obtained by taking moment of compressive force about the point where compressive force acts. The lever arm is the perpendicular distance between the total compression and total tensile force (Refer fig. 1.23(a) and (b))

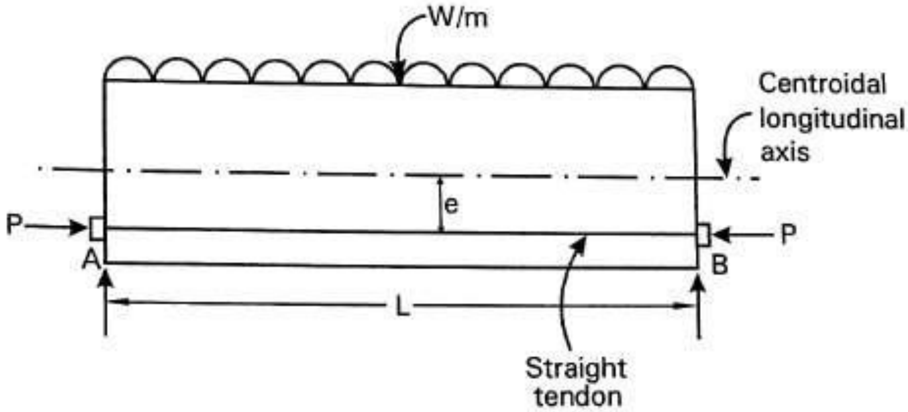
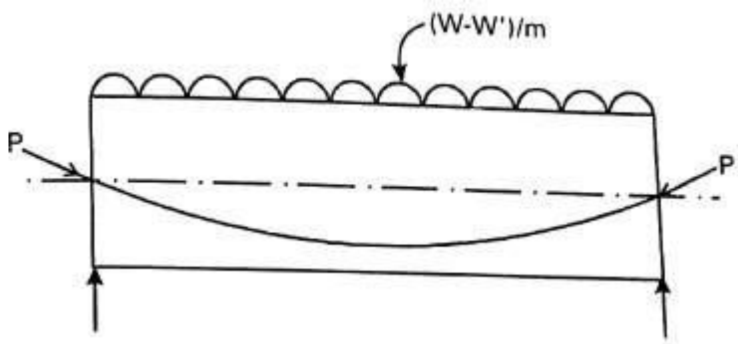


Fig. 1.23 (a) : Psc beam with straight tendon

Load Balancing Concept



If W is the external applied uniformly distributed load (*udl*) acting downwards and W' is the upward *udl* due to pretesting force, then net load on the beam is $(W - W')$. If $W = W'$, then the beam is subjected to only axial prestress, and the bending curvature is zero. If the anchorage is located at the centroid of the section, then the beam is subjected to uniform prestress. The beam does not deflect. If $W \neq W'$, then the BM due to unbalanced load is calculated. The resulting flexural stress distribution is calculated using the flexural formula $\left(\frac{My}{I}\right)$. This is added to the axial stress, ie., $\left(P/A\right)$

We can understand by inspection that the cable profile is the mirror image of the BMD. The location of the cable from the centroidal axis is given as follows:

$$M = Pe$$

$$\Rightarrow e = \frac{M}{P}$$

This is been satisfied by providing the cable profile which reflects the BMD

Cracking Moment

It means the BM at which visible cracks appear in prestressed concrete members.

When prestress is transferred to concrete compressive stresses are developed at the soffit of the beam. Then the beam is subjected to transverse load, the compressive stress at soffit is decreaing and finally reach zero.

On further loading the beam tensile stresses are developed at the soff it of the beam. Microscopic cracks redeveloped in beam due to weak in tension of concrete.

As the load is further increase the cracks are visible in the tension zone. The crack width at this stage is 0.01 to 0.02mm

The tensile stress, which will develop the visible cracks at the soffit of the beam is depends on the

- Quality of reinforcement
- Location of reinforcement
- Quality of concrete

It is generally consider the visible cracks are occurs, when the tensile stress in concrete reaches a value equal to the modulus of rupture of concrete.

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MODULE - 2

LOSSES OF PRESTRESS IN BEAMS

Elastic Shortening (ES)

Shortening in steel that occurs as soon as Fi is transferred to the concrete member and the member as a whole shortens.

Fi = Pre-stress just before transfer

F = Final stress after losses

Fo = immediately after transfer – very difficult to estimate

Note: The value of Fo may not be known, but it is not necessary, as the losses from Fi to Fo is only a small percentage of Fi. Total accuracy is relative anyway, as Ec – the young's modulus of concrete – cannot be determined accurately.

Therefore

$$ES = E_s\delta$$

where δ is the shortening in steel that occurs as soon as F_i is transferred to the concrete member and the member as a whole shortens. Thus δ is the shortening in the member due to F_i at the level of steel.

$$\delta = \frac{f_c}{E_C}$$

$$= \frac{F_O}{A_C E_C}$$

Since f_c is the stress in concrete at level of steel and is $\frac{F_O}{A_C}$

$$ES = \frac{F_O}{A_C E_C} E_S$$

$$ES = \frac{nF_i}{A_T}$$

$\therefore$ whichever way the ES is calculated

ES = n (concrete stress at level of steel)

If external loads are acting on the member, then concrete, then concrete stress due to all loads at level of steel must be determined.

$$f_c = -\frac{F_O}{A_G} - \frac{F_O e^2}{I} + \frac{M_G e}{I}$$

Note: A_G, the gross-area, instead of the transformed sectional area, leads to simpler calculations and fairly accurate results.

$F_o \approx 0.9F_i$ for pre-tensioned member

$$f_c = -\frac{F_O}{A_G}$$

$$ES = nf_c$$

Creep

Among the many factors affecting creep are volume to surface ratio, age of concrete at time of pre- stress, relative humidity, type of concrete (lightweight / normal). Creep is assumed to occur in the member after permanent loads are imposed after pre-stress. Creep occurs over a long period of time under sustained load. Part of initial compressive strain induced in concrete immediately after transfer is reduced by the tensile strain produced by superimposed permanent loads.

Therefore for bonded members, loss due to creep

$$CR = \theta n(f_{cir} - f_{cds})f_c$$

$$n = \frac{E_S}{E_C}$$

θ = Creep coefficient – Cl 4.5.3 & Cl 5.2.5.1

f_{cir} = concrete stress at level of steel immediately after transfer.

f_{cds} = stress in concrete at steel level due to superimposed dead loads applied to the member after transfer of pre-stress

Shrinkage of Concrete

Factors like volume to surface ratio, relative humidity, time from end of moist curing to application of pre-stress, affect shrinkage in concrete. Shrinkage is time-dependant and about 80% of the final loss due to shrinkage occurs in the first year and 100% after several years.

Shrinkage strain

$\varepsilon_{sh} = 0.0003$ for pretensioned member

$= \dfrac{0.0002}{\log_{10}(t+2)}$ for posttensioned member and Cl 5.2.4.1

may be increased by 50% in dry condition but not more than 0.0003

Relaxation of Steel

When elongation is sustained over pre-stressing cable for a long time, the pre-stress will decrease gradually. The RE – loss due to relaxation depends on type of steel, time, as well as the ratio of $\dfrac{f_i}{f_p}$ where f_i is the initial pre-stress and f_p is the characteristic strength of steel.

RELAXATION LOSSES FOR PRESTRESSING STEEL AT 1 000 H AT 27°C INITIAL STRESS RELAXATION	
INITIAL STRESS	RELAXATION LOSS N/mm2
0.5 *fp*	0
0.6 *fp*	35
0.7 *fp*	70
0.8 *fp*	90

Anchorage Slip

In post-tensioning, when the jack is released, the full pre-stress is transferred to the anchorage and they tend to deform, allowing the tendon to slacken. Friction wedges will slip a little before they grip the wire firmly. So, in post-tensioning the wedges are positively engaged before the jack is released. In pre- tensioning also, the anchorage slip is compensated for during stressing operation. The loss is caused by a fixed shortening of the anchorages, so the percentage loss is higher in shorter wires than in long ones.

Frictional Loss

Frictional loss comprise of two parts: (1) The length effect and (2) The curvature effect.

The length effect or the wobble effect of the duct is the friction that will exist between straight tendon and the surrounding material. This loss is dependent on the length and stress in the tendon, the coefficient of friction between the contact materials, the workmanship and the method used in aligning and obtaining the duct.

The curvature effect is the loss due to intended curvature of the tendon. This again depends on the coefficient of friction between the materials and the pressure exerted by the tendon on the curvature.

For un-bonded tendon, lubrication, in the form of grease and plastic tube wrapping can be used to advantage. For bonded tendon lubricant in the form of water soluble oils are used during stressing operation and flushed off with after before grouting. Jacking from both ends of the beam will also reduce loss due to friction.

For straight or moderately curved structures, with curved or straight cables, the value of pre-stressing force Px at a distance x meters from tensioning end and acting in the direction of the tangent to the curve of the cable, shall be calculated as below:

Px = Poe– (μα + kx)

Where Po = pre-stressing force in the pre-stressed steel at the tensioning end acting in the direction of the tangent to the curve of the cable, α = cumulative angle in radians through which the tangent to the cable profile has turned between any two points under consideration, μ = coefficient of friction in curve; unless otherwise proved by tests, μ may be taken as: 0.55 for steel moving on smooth concrete, 0.30 for steel moving on steel fixed to duct, and 0.25 for steel moving on lead, k = coefficient for wobble or wave effect varying from 15 × 10–4 to 50 × 10–4 per meter. The expansion of the equation for Px for small values of (μα + kx) may be Px = Po (1 – μα – kx).

Problems

1. A post-tensioned concrete beam, simply supported over a span of 12 m is of cross-section 230 × 750 mm and is prestressed with 10 numbers of 7 mm diameter parabolic cable bars with zero eccentricity at the support and 200 mm at midspan. Calculate the loss due to different causes for the following data.

Grade of concrete = M40

Initial prestress = 1000 N/mm²

Coefficient of curvature effect, μ = 0.50

k = 0.003/m (Wobble coefficient)

Anchorage slip = 5 mm at jacking end

Creep coefficient = 1.6, Shrinkage of concrete = 0.0002 = ε_{sh}

Relaxation of steel stress = 3%, E_S = 210 kN/mm², E_C = 37.50 kN/mm²

Calculate the total percentage of loss and the jacking force required.

Solution :

Given, $f_{ck} = 40$ N/mm², $P_o = P_i = 1000$ N/mm², $\mu = 0.50$, $k = 0.003$/m, $\phi = 1.6$,

$E_s = 210$ kN/mm², $E_C = 37.50$ kN/mm², $b = 230$ mm, $D = 750$ mm, $L = 12$ m, $e = 200$ mm

$$\text{Moment of inertia, } I = \frac{bD^3}{12} = \frac{230 \times 750^3}{12} = 8.08 \times 10^9 \text{ mm}^4$$

Prestressing force = No. of wires × Area of wire × initial prestress

$$\therefore \text{ Prestressing force, } P = 10 \times \frac{\pi}{4} \times 7^2 \times 1000 = 384.84 \text{ kN}$$

$$\text{Loss due to slip} = \frac{E_S \Delta}{L}, \quad \Delta = 5 \text{ mm (Given)}$$

$$= \frac{210 \times 10^3 \times 5}{12000} = 87.5 \text{ N/mm}^2$$

Loss due to friction = $P_o\,(\mu\alpha + kx)$

α = Total change in slope from one end to the other end

$$\therefore \quad \alpha = 2\theta \text{ or } 2 \times \frac{4e}{L}$$

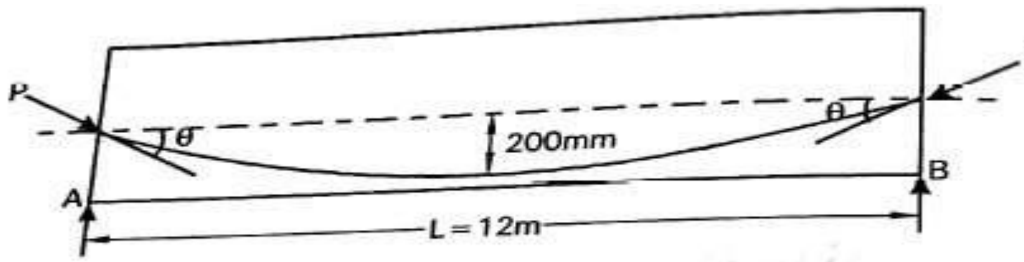


Fig. 2.2(a) : Details of PSC beam

$$\alpha = \frac{2 \times 4 \times 200}{12000} = 0.133, \qquad x = L = 12 \text{ m}$$

$$\text{Loss due to friction} = 1000 \times (0.5 \times 0.133 + 0.003 \times 12)$$

$$= 102.5 \text{ N/mm}^2$$

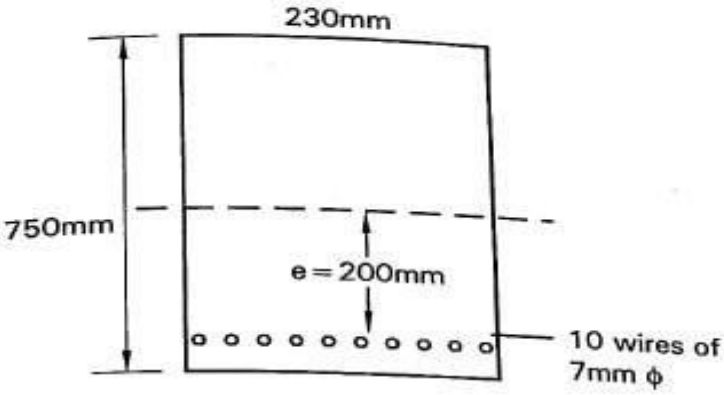


Fig. 2.2(b) c/s of beam

$$\text{Loss due to creep} = \phi m f_c$$

$$\text{Modular ratio, } m = \frac{E_S}{E_C} = \frac{210}{37.5} = 5.6$$

$$f_c = \text{Original or actual prestress in concrete} = \frac{P}{A} + \frac{Pe'^2}{I}$$

$$e' = \text{Average eccentricity}$$

$$= \frac{2}{3} \times e = \frac{2}{3} \times 200 \text{ — For parabolic cable}$$

$$e' = 133.33 \text{ mm}$$

$$\text{Area} = A = b \times D = 230 \times 750 = 172.5 \times 10^3 \text{ mm}^2$$

$$f_c = \frac{384.84 \times 10^3}{172.5 \times 10^3} + \frac{384.84 \times 10^3 \times (133.33)^2}{8.08 \times 10^9}$$

$$= 2.23 + 0.84$$

$$f_c = 3.07 \text{ N/mm}^2$$

Loss due to creep $= 1.6 \times 5.6 \times 3.07 = 27.50 \text{ N/mm}^2$

Loss due to shrinkage = Shrinkage strain of concrete × Modulus of elasticity of steel

$$= \varepsilon_{sh} \times E_s$$

$$= 0.0002 \times 210 \times 10^3 = 42 \text{ N/mm}^2$$

Loss due to relaxation $= \frac{3}{100} \times$ Initial prestress

$$= \frac{3}{100} \times 1000 = 30 \text{ N/mm}^2$$

A simply supported post-tensioned concrete beam of span 15 m has a rectangular cross section 300 × 800 mm. The prestress at ends is 1300 kN with zero eccentricity at the supports and 250 mm at the centre the cable profile being parabolic. Assuming $k = 0.15$ per 100 metres and $\mu = 0.35$. Determine the loss of stress due to friction at the centre of the beam.

Moment of inertia of the beam section

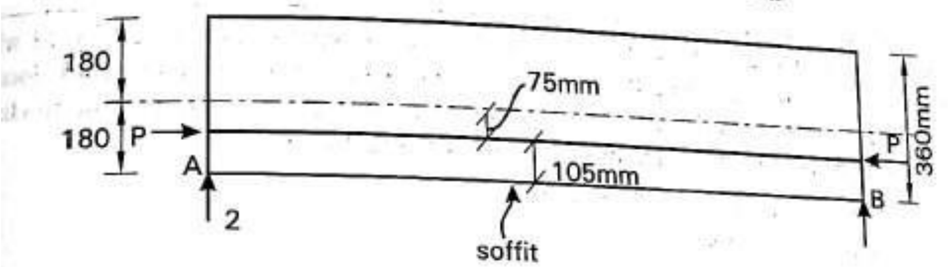
$$I = \frac{250 \times 360^3}{12} = 9.72 \times 10^8 \text{ mm}^4$$

Eccentricity $\quad e = 180 - 105 = 75 \text{ mm}$

Modular ratio, $\quad m = \frac{E_s}{E_c} = \frac{210}{36.85} = 5.70$

Area of the beam section

$$A = 250 \times 360 = 90000 \text{ mm}^2 = 9 \times 10^4 \text{ mm}^2$$



Area of the steel wires $= A_s = 10 \times \frac{\pi}{4} \times 8^2 = 502.65\ \text{mm}^2$

Initial prestressing force $= P_I = 1000 \times 502.65 = 502655$ N or 502.655 kN

Stress in concrete at the level of steel $= \frac{P_I}{A} + \frac{P_I e'^2}{I}$; $e' = e = 75$ mm — straight tendon

$$f_c' = \frac{502655}{9 \times 10^4} + \frac{502655 \times 75^2}{9.72 \times 10^8} = 5.59 + 2.91 = 8.50\ \text{N/mm}^2$$

Loss of stress due to elastic shortening of concrete $= mf_c' = 5.70 \times 8.50 = 48.45\ \text{N/mm}^2$

$\therefore$ Stress in the wires $= 1000 - 48.45 = 951.55\ \text{N/mm}^2$

Prestressing force $P = 951.55 \times 502.65 = 478296.6\ \text{N/mm}^2$

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MODULE - 3

DESIGN OF SECTIONS FOR FLEXURE

Permissible Stresses For Flexure Member

Steel: Steel stress for pre-tensioned tendons immediately after transfer or post-tensioned tendons after anchorage is:

$$f_{pi} = 0.87 f_{pu}$$

Where f_{pi} = Maximum initial pre-stress, and f_{pu} = Ultimate tensile stress in tendon.

Concrete in Compression

Concrete stress after **transfer and before losses** in extreme fiber

Compression = 0.54 fck to 0.37 fck (for M30 to M60) for **Post-Tension**

= 0.51 fck to 0.44 fck (or M40 to M60) for **Pre-Tension**

Concrete stress **at service loads** after transfer and **after losses** in extreme fiber

Compression = 0.41 fck to 0.35 fck (for M30 to M60) for **Post-Tension**

= 0.34 fck to 0.27 fck (or M40 to M60) for **Pre-Tension**

Concrete in Tension

Concrete stress after transfer and before losses in extreme fiber

1. For Type 1 members, Tension = 0.
2. For Type 2 members, Tension = 3.0 MPa to 4.5 MPa
3. For Type 3 members, Tension = 4.1 MPa to 4.8 MPa

Problems

4. Find the ultimate moment of resistance of the pre-tensioned beam section of width 300 mm and effective depth 600 mm for the following data :
f_{ck} = 40 N/mm², f_p = 1500 N/mm², A_p = 500 mm², f_{pu} = 0.87 f_p

Solution :

Given, $b = 300$ mm, $d = 600$ mm, $f_{ck} = 40$ N/mm², $f_p = 1500$ N/mm²

$A_p = 500$ mm², $f_{pu} = 0.87 f_p$

➢ The effective reinforcement ratio

$$\frac{A_p f_p}{b d f_{ck}} = \frac{500 \times 1500}{300 \times 600 \times 40} = 0.104$$

Given

$$f_{pu} = 0.87 f_p = 0.87 \times 1500 = 1305 \text{ N/mm}^2$$

From table 11 of IS:1343 for pre-tensioned member

$$\frac{x_u}{d} = 0.217 \Rightarrow x_u = 0.217 \times d = 0.217 \times 600 = 130.2 \text{ mm}$$

➢ Ultimate moment of resistance (IS:1343-1980, Appendix - B)

$$M_u = f_{pu} A_p (d - 0.42 x_u)$$

$$= 1305 \times 500 (600 - 0.42 \times 130.2) = 355.82 \times 10^6 \text{ N.mm}$$

$$\therefore M_u = 355.82 \text{ kN.m}$$

A post-tensioned beam with unbonded tendons is of rectangular cross-section 500 mm × 1000 mm. The cross-sectional area of prestressing steel is 3000 mm². The effective prestress after considering all losses is 1000 Mpa. The effective span of the beam made M40 concrete is 15 m. Estimate the ultimate moment of resistance of the section using codal provisions.

Solution :

Given, post-tensioned beam with unbonded tendon, $b = 500$ mm, $d = 1000$ mm,

$A_p = 3000$ mm², $f_p = 1000$ Mpa

i.e., $f_p = 1000$ N/mm², $f_{ck} = 40$ N/mm², $L = 15$ m

➤ Calculation of effective reinforcement and $\frac{L}{d}$ ratio

➤ Calculation of effective reinforcement and $\frac{L}{d}$ ratio. Refer table 12 of IS:1343-1980

$$\frac{A_p f_p}{f_{ck} bd} = \frac{2840 \times 900}{40 \times 400 \times 800} = 0.199 \approx 0.20$$

$$\frac{L}{d} = \frac{16 \times 10^3}{800} = 20$$

➤ Find f_{pu} and x_u using table 12 of IS:1343 - 1980

For

$$\frac{A_p f_p}{f_{ck} bd} = 0.20 \text{ and } \frac{L}{d} = 20$$

$$\frac{f_{pu}}{f_p} = 1.16 \Rightarrow f_{pu} = 1.16 \times f_p = 1.16 \times 900 = 1044 \text{ N/mm}^2$$

$$\frac{x_u}{d} = 0.58 \Rightarrow x_u = 0.58 \times 800 = 464 \text{ mm}$$

Calculation of ultimate moment of resistance

$$M_u = f_{pu} A_p (d - 0.42\, x_u)$$

$$= 1044 \times 2840\,(800 - 0.42 \times 464)$$

$$= 1794.15 \times 10^6 \text{ N.mm}$$

$$\therefore\ M_u = 1794.15 \text{ kN.m}$$

Find the ultimate moment of resistance of unbonded post-tensioned beam section of width 300 mm and effective depth 600 mm for the following data :

f_{ck} = 40 N/mm², f_p = 1500 N/mm², A_p = 500 mm²

Solution :

Given, $b = 300$ mm, $d = 600$ mm, $f_{ck} = 40$ N/mm², $f_p = 1500$ N/mm²

$A_p = 500$ mm² and $L = 10$ m

➤ Calculation of effective reinforcement and $\frac{L}{d}$ ratio (Refer table 12 of 1343-1980)

$$\frac{A_p f_p}{f_{ck} bd} = \frac{500 \times 1500}{40 \times 300 \times 600} = 0.10$$

$$\frac{L}{d} = \frac{10 \times 10^3}{600} = 16.67$$

Refer IS:1343-1980, table 11

$$\left(\frac{f_{pu}}{0.87 f_p}\right) = 1.0$$

$$f_{pu} = (1.0 \times 0.87 \times 1600) = 1392 \text{ N/mm}^2$$

∴

By interpolation, we get,

$$\left(\frac{x_u}{d}\right) = 0.5$$

and

$$x_u = (0.5 \times 600) = 300 \text{ mm}$$

∴

➤ Ultimate flexural strength of the section

$$M_u = A_p f_{pu} [d - 0.42\ x_u]$$

$$= 1054.51 \times 1392\ (600 - 0.42 \times 300) = 695.77 \text{ kN.m}$$

A pre-tensioned beam of rectangular section 300 wide by 700 mm deep is stressed by 800 mm² of high tensile steel located at effective depth of 600 mm. The beam is also reinforced with supplementary reinforcements consisting of 4 bars of 25 mm diameter of Fe-415 grade HYSD steel, located 100 mm from the soffit. Estimate the flexural strength of the section. Assume the ultimate tensile strength of tendons as 1600 N/mm² and the characteristic cube strength of concrete as 40 N/mm².

Solution :

b = 300 mm $\quad d = 600$ mm

f_{ck} = 40 N/mm² $\quad f_p = 1600$ N/mm²

f_y = 415 N/mm² $\quad A_{p_1} = 800$ mm²

$$A_s = 4 \times \frac{\pi}{4} \times 25^2 = 1963.5 \text{ mm}^2$$

The untensioned supplementary reinforcement is replaced by an equivalent area of prestressing steel given by the relation

$$A_{pe} = \left[\frac{A_s f_y}{f_p}\right] = \left[\frac{1963.5 \times 415}{1600}\right] = 509.28 \text{ mm}^2$$

$\therefore$ Total area of prestressing steel = A_p = (800 + 509.08) = 1309.28 mm²

- Effective reinforcement ratio

$$\left(\frac{A_p f_p}{f_{ck} bd}\right) = \left(\frac{1600 \times 1309.28}{40 \times 300 \times 600}\right) = 0.29$$

Refer IS:1343-1980, table 11

$$\left(\frac{f_{pu}}{0.87 f_p}\right) = 1 \text{ and } \left(\frac{x_u}{d}\right) = 0.63$$

$$\Rightarrow \quad f_{pu} = (1 \times 0.87 \times 1600) = 1392 \text{ N/mm}^2$$

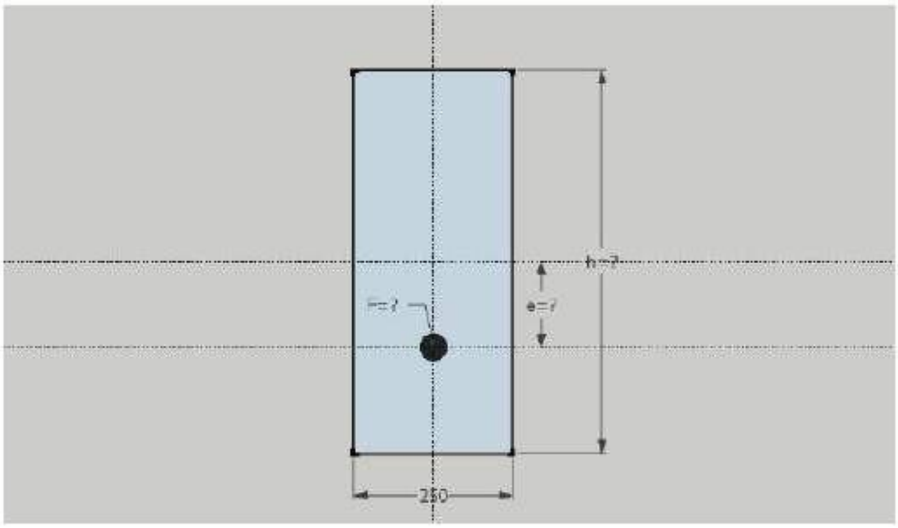
$$\Rightarrow \quad x_u = (0.63 \times 600) = 378 \text{ mm}$$

- Calculate ultimate moment of resistance

$$M_u = f_{pu} A_p (d - 0.42\, x_u)$$

$$= (1392 \times 1309.28)(600 - 0.42 \times 378) = 804.16 \text{ kN}$$

Design a post-tensioned beam of l_e = 12 m to carry a live load of 12 kN/m throughout its length. The width of beam b = 250 mm. $f_{ct} = f_{cw} = -17 MPa$ and $f_{tt} = f_{tw} = 1.4 MPa$. η = 0.85.



Assume depth of beam = h mm

A = 250h mm²

$M_G = \frac{\left(0.25 \times \frac{h}{1000}\right) \times 24 \times 12^2}{8}$ = 0.108h kN-m

$M_L = \frac{12 \times 12^2}{8}$ = 216 kN-m

Min Z is governed by Z_b. From Eq.4

$f_{cr} = f_{tw} - \eta f_{ct}$

$f_{cr} = 1.4 - 0.85(-17)$ = 15.85 MPa

$Z_b = \frac{M_G(1-n) + M_L}{f_{cr}} \quad K(6)$

$= \frac{(0.108h)(1-0.85) \times 10^6 + 216 \times 10^6}{15.85} = \frac{10^6(216 + 0.0162h)}{15.85}$

Z_b also $= \frac{250h^2}{6}$

From which

h = 580 mm

A = 250 x 580 = 145×10^3 mm^2

$Z_t = Z_b = Z = \frac{250 \times 580^2}{6}$ = 14×10^6 mm^3

M_G = 62.64 kN-m

$f_t = f_{tt} + \frac{M_G}{Z_t}$

$= 1.4 + \frac{62.64 \times 10^6}{14 \times 10^6}$ = 5.87 MPa

$f_b = \frac{1}{\eta}\left(f_{tw} - \frac{M_G + M_L}{Z_b}\right)$

$$= \frac{1}{0.85}\left(1.4 - \frac{(62.64+216)\times10^6}{14\times10^6}\right)$$ = -21.76 MPa

$$\frac{F_i}{A} = \frac{-f_b Z_b - f_t Z_t}{Z_b + Z_t} \ldots(9)$$

$$= \frac{(21.76-5.87)\times14\times10^6}{2\times(14\times10^6)}$$ = 7.945

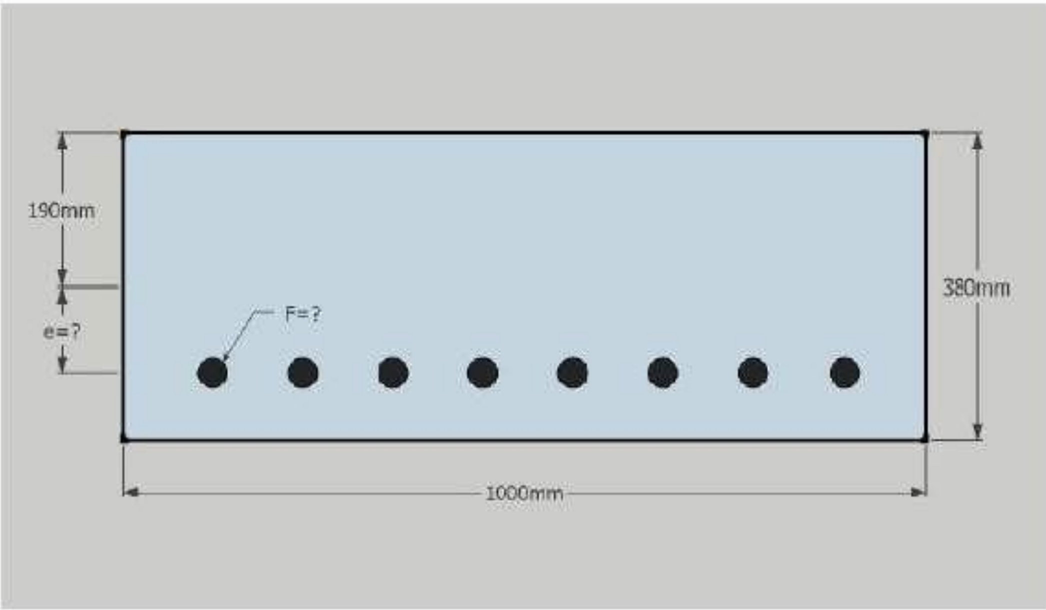
Fi = 7.945 x 145 x10^3 = 1152 kN

$$Ae = \frac{Z_b Z_t (f_b - f_t)}{f_t Z_t + f_b Z_b} \ldots(10)$$

$$= \frac{14\times14\times10^{12}(-21.76-5.87)}{14\times10^6(5.87-21.76)}$$ = 24.3436x10^6

e = 167.89 mm

A post-tensioned concrete bridge slab of l_e = 10 m is 380 mm thick. It is stressed with parallel cables stressed to 360 kN each. w_L = 25 kN/m2. Losses are 20%. $f_{tt} = f_{tw} = 0.7MPa$. Calculate the e_{max} and spacing of cable at mid-span.



η = 0.80

A = 1000 X 380 = 380000 mm²

I $= \frac{1000 \times 380^3}{12}$ = 4572.66 x10⁶ mm⁴

$Z_t = Z_b = Z$ $= \frac{4572.66 \times 10^6}{\left(\frac{380}{2}\right)}$ = 24.07 x10⁶ mm³

w_s = 1 x 0.38 x 24 = 9.12 kN/m

M_G $= \frac{9.12 \times 10^2}{8}$ = 114 kN-m

M_L $= \frac{25 \times 10^2}{8}$ = 312.50 kN-m

$\frac{M_G}{Z}$ $= \frac{114 \times 10^6}{24.07 \times 10^6}$ = 4.74

$\frac{M_L}{Z}$ $= \frac{312.5 \times 10^6}{24.07 \times 10^6}$ = 12.98

At mid-span

$$f_t = f_{tt} + \frac{M_G}{Z_t}$$

$= 0.7 + 4.74$ = 5.44 MPa

$$f_b = \frac{1}{\eta}\left(f_{tw} - \frac{M_G + M_L}{Z_b}\right)$$

$= \frac{1}{0.8}(0.7 - (4.74 + 12.98))$ = -21.275 MPa

$$\frac{F_i}{A} = \frac{-f_b Z_b - f_t Z_t}{Z_b + Z_t} \quad (9)$$

$= \frac{(21.275 - 5.44) \times 24.07 \times 10^6}{2 \times (24.07 \times 10^6)}$ = 7.9175

Fi = 7.9175 x 380 x10^3 = 3008.65 kN

$$Ae = \frac{Z_b Z_t (f_b - f_t)}{f_t Z_t + f_b Z_b} \ldots (10)$$

$$= \frac{24.07 \times 24.07 \times 10^{12} (-21.275 - 5.44)}{24.07 \times 10^6 (5.44 - 21.275)}$$ = 40.6081 x10^6

e mid-span = 106.86 mm

At support

$M_G = M_L = 0$

e at support = 68.944

At mid-span, the stress at top and bottom at transfer and working load are respectively.

$$-\frac{F_i}{A} + \frac{F_i e}{Z_t} - \frac{M_G}{Z_t} = f_{tt} \ldots (1)$$

$$-\frac{F_e}{A} - \frac{F_e e}{Z_b} + \frac{M_G}{Z_b} + \frac{M_L}{Z_b} = f_{tw} \ldots (4)$$

ie. $$-\eta \frac{F_i}{A} - \eta \frac{F_i e}{Z_b} + \frac{M_G}{Z_b} + \frac{M_L}{Z_b} = f_{tw} \ldots (4)$$

Multiplying Eq.1 by η and adding it to Eq.4 above, and remembering $Z_t = Z_b = Z$,

$$-2\eta \frac{F_i}{A} + (1-\eta) \frac{M_G}{Z} + \frac{M_L}{Z} = f_{tw} + \eta f_{tt}$$

$$-2 \times 0.8 \frac{F_i}{A} + (1 - 0.8) 4.74 + 12.98 = 0.7 + 0.8 \times 0.7$$ from which

F_i = 3009 kN

Likewise, multiplying Eq.1 by η and subtracting it from Eq.4 above,

$$-2\eta\,\frac{F_i e}{Z}+(1+\eta)\,\frac{M_G}{Z}+\frac{M_L}{Z}=f_{tw}-\eta f_{tt}$$

$$-2\times 0.8\,\frac{F_i e}{A}+(1+0.8)4.74+12.98=0.7-0.8\times 0.7$$

e at mid-span = 106.358 mm

At support

$M_G = M_L = 0$

$$-\frac{F_i}{A}+\frac{F_i e}{Z_t}-0=f_{tt}\,\mathrm{K}(1)$$

$$-\frac{3009\times 10^6}{380000}+\frac{3009\times 10^6\times e}{24.07\times 10^6}=0.7$$

e at support = 68.944

Spacing of cables

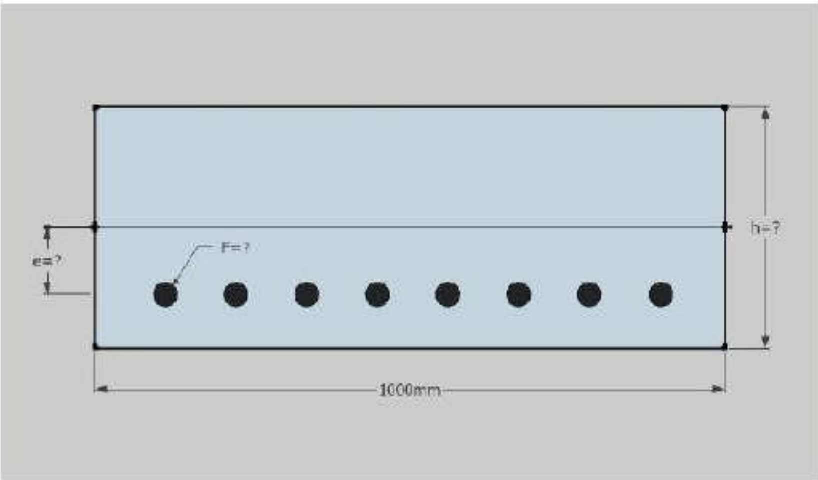
F_i = 3009 kN

Force per cable = 360 kN

No of cables $= \dfrac{3009\times 10^3}{360\times 10^3}$ $\approx$ 9 Nos

Spacing = $\dfrac{1000}{9}$ = 112 mm c/c

A post-tensioned concrete one-way bridge slab of l_e = 10 m is stressed with parallel cables stressed to 500 kN each. w_L = 25 kN/m2. Losses are 20%. $f_{ct} = f_{cw} = -15MPa$ and $f_{tt} = f_{tw} = 0$.



η	= 0.80
Assume depth of slab	= h mm
Width of slab	= 1000 mm
A	= 1000h mm^2

M_G = $\frac{\left(1\times\frac{h}{1000}\right)\times 24\times 10^2}{8}$ = 0.3 kN-m

M_L = $\frac{25\times 10^2}{8}$ = 312.5 kN-m

Min Z is governed by Z_b. From Eq.4

$$f_{cr} = f_{tw} - \eta f_{ct}$$

f_{cr} = $0-0.8(-15)$ = 12 MPa

$$Z_b = \frac{M_G\ (1-n)+M_L}{f_{cr}} \ldots(6)$$

= $\frac{(0.3h)(1-0.8)\times 10^6 + 312.5\times 10^6}{12}$ = $\frac{10^6(312.5+0.06h)}{12}$

Z_b also $=\dfrac{100h^2}{6}$

From which

h = 410 mm

A = 1000 x 410 = 410000 mm^2

$Z_t = Z_b = Z$ $=\dfrac{1000\times 410^2}{6}$ = 28.02x10^6 mm^3

M_G = 123 kN-m

$$f_t = f_{tt} + \frac{M_G}{Z_t}$$

$$= 0+\frac{123\times10^6}{28.02\times10^6}$$ = 4.39 MPa

$$f_b = \frac{1}{\eta}\left(f_{tw} - \frac{M_G + M_L}{Z_b}\right)$$

$$= \frac{1}{0.8}\left(0-\frac{(123+312.50)\times10^6}{28.02\times10^6}\right)$$ = -91.43 MPa

$$\frac{F_i}{A} = \frac{-f Z - f Z}{Z_b + Z_t} \ldots(9)$$

$$= \frac{19.43-4.39}{2}$$ = 7.52

Fi = 7.52 x 410000 = 3083.20 kN

$$Ae = \frac{Z_b Z_t (f_b - f_t)}{f_t Z_t + f_b Z_b} \ldots(10)$$

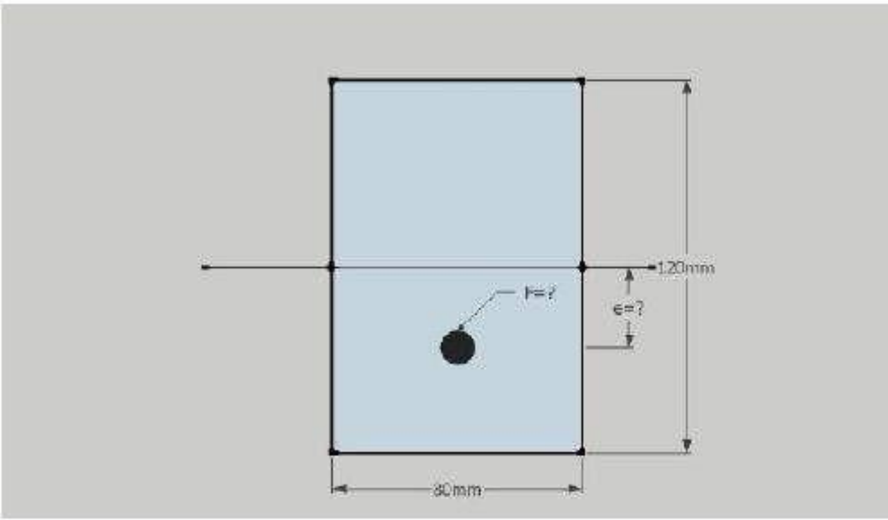
$$= \frac{28.0210^6(-19.43-4.39)}{(4.39-19.43)}$$ = 443.774 x10^6

e mid-span = 108.24 mm

No of cables $= \frac{3084 \times 10^3}{500 \times 10^3}$ ≈ 7 Nos

Spacing = $\frac{1000}{7}$ = 143 mm c/c

A pre-tensioned simply supported beam of size 80 mm x 120 mm and le = 3 m caries two 4 kN loads at third points along the span. Losses are 20%. $f_{tt} = 0, f_{tw} = 1.4MPa$. Design the beam with 3mm wires for $f_i = 1400$ MPa each.



η = 0.80

A = 80 X 120 = 9600 mm^2

I $= \frac{80 \times 120^3}{12}$ = 11.52 x10^6 mm^4

$Z_t = Z_b = Z$ $= \frac{11.52 \times 10^6}{\left(\frac{120}{2}\right)}$ = 0.192 x10^6 mm^3

w_s = 0.08 x 0.12 x 24 = 0.23 kN/m

M_G $= \frac{0.23 \times 3^2}{8}$ = 0.2592 kN-m

M_L $= 4 \times 1$ = 4.0 kN-m

$\frac{M_G}{Z}$ = $\frac{0.2592\times10^6}{0.192\times10^6}$ = 1.35

$\frac{M_L}{Z}$ = $\frac{4.0\times10^6}{0.192\times10^6}$ = 20.83

At mid-span

$$f_t = f_{tt} + \frac{M_G}{Z_t}$$

= $0+1.35$ = 1.35 MPa

$$f_b = \frac{1}{\eta}\left(f_{tw} - \frac{M_G + M_L}{Z_b}\right)$$

= $\frac{1}{0.8}(1.4-(1.35+20.83))$ = -25.975 MPa

$$\frac{F_i}{A} = \frac{-f_b Z_b - f_t Z_t}{Z_b + Z_t} \ldots (9)$$

= $\frac{(25.975-1.35)}{2}$ = 12.3125

Fi = 12.3125 x 9600 = 118.20 kN

$$Ae = \frac{Z_b Z_t (f_b - f_t)}{f_t Z_t + f_b Z_b} \ldots (10)$$

= $\frac{0.192\times10^6(-25.975-1.35)}{(1.35-25.975)}$ = 0.213x10^6

e = 22.193 mm

A_w = Area of one wire

= $\frac{\pi\times3^2}{4}$ = 7.07 mm^2

f_i in one wire = 7.07 x 1400 = 9.896 kN

No of cables $= \dfrac{118.20 \times 10^3}{9.896 \times 10^3}$ $\approx$ 12 Nos

Spacing = $\dfrac{1000}{9}$ = 112 mm c/c

An unsymmetrical I section has the following sectional property: h = 1000 mm, A = 345 000 mm^2, Z_t = 95 x10^6 mm^3, Z_b = 75 x 10^6 mm^3, cgc = 440 mm from top, M_G = 1012 kN-m, M_L = 450 kN-m. Design the section if $f_{ct} = f_{cw} = -15 MPa$ and $f_{tt} = f_{tw} = 0$. η = 0.85

$$f_t = f_{tt} + \frac{M_G}{Z_t}$$

$$= 0 + \frac{1012 x 10^6}{95 \times 10^6} \qquad = 10.65 \text{ MPa}$$

$$f_b = \frac{1}{\eta}\left(f_{tw} - \frac{M_G + M_L}{Z_b} \right)$$

$$= \frac{1}{0.85}\left(0 - \frac{(1012 + 450) \times 10^6}{75 \times 10^6} \right) \qquad = -22.93 \text{ MPa}$$

$$\frac{F_i}{A} = \frac{-f_b Z_b - f_t Z_t}{Z_b + Z_t} \quad \text{K}(9)$$

$$= \frac{22.93 \times 75 - 10.65 \times 95}{75 + 95} \qquad = 4.1647$$

Fi = 4.1647 x 345000 = 1436.82 kN

$$Ae = \frac{Z_b Z_t (f_b - f_t)}{f_t Z_t + f_b Z_b} \quad \text{K}(10)$$

$$= \frac{75 \times 95 \times 10^{12} (-22.93 - 10.65)}{(10.65 \times 95 - 22.93 \times 75) \times 10^6} \qquad = 361.339 \text{ x}10^6$$

e = 1047.36 mm

e_{avil} $= y_b - cover$

$= (1000\text{-}440) - 100$ $= 460$ mm

For this e_{avil}, the F_i required is:

$$F_i = \left(\frac{f_b A Z_b}{Z_b + Ae} \right) \ldots (11)$$

$$F_i = \frac{22.93 \times 345000 \times 75 \times 10^6}{75 \times 10^6 + 345000 \times 460} = 2538.78 \text{ kN}$$

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MODULE - 4

DESIGN FOR SHEAR

Introduction

In the case of reinforced concrete beams the designed for shear was made on the assumption that the failure is liable to be caused by diagonal tension i.e., the failure was taken to be due to principal tensile stresses. But, in the case of pre-stressed concrete members, however, due to the introduction of compressive stresses by prestressing, the principle tensile stresses are reduced. Further if the cable are inclined or curved, the vertical components of the tensions in the cables also will resist shear.

Types of Shear cracks:

The types and formation of cracks depends on the span-to-depth (L/d) ratio of the beam and loading. These variables influence the moment and shear along the length of the beam. For a simply supported beam under uniformly distributed load, without prestressing, three types of cracks are identified.

1. **Flexural cracks:** These cracks form at the bottom near the midspan and propagate upwards.

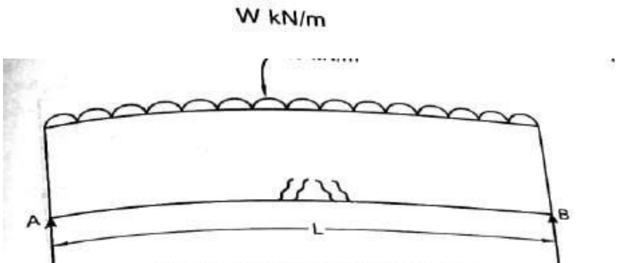


Fig. 4.2 : Initiation of flexural cracks

2. **Web shear cracks :** These cracks form near the neural axis close to the support and propagate inclined to the beam axis.

The ultimate shear resistance of prestressed concrete sections with web-shear cracking but without flexural cracks is mainly governed b limiting value of the principal tensile stress developed in concrete. The failure is assumed to take place when the principal tension exceeds the tensile strength of the concrete. Refer IS: 1343-1980, clause 22.4, the ultimate shear resistance of a section uncracked in flexure, $V_c = V_{co}$, is given by:

$$V_{co} = 0.67\, bD \sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

3. Flexure shear cracks:

These cracks form at the bottom due to flexure and propagate due to both flere and shear.

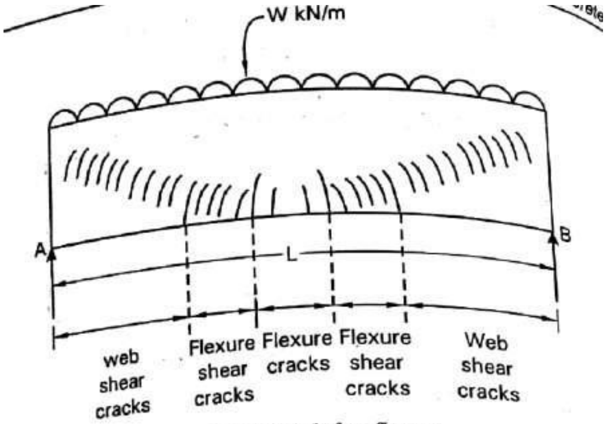


Fig. 4.4 : Cracks before flexure

The recommendations of IS: 1343-1980, clause 22.4.2 are similar for the computation of the ultimate shear resistance V_{cr} of sections cracked in flexure, $V_c = V_{co}$, which is expressed as,

$$V_{cr} = \left(1 - 0.55\frac{f_{pe}}{f_p}\right)\zeta_0\, bd + M_0 \frac{V}{M}$$

V and M = shear force and bending moment respectively, at the section considered due to ultimate loads.

V_{cr} should be taken as not less than 0.1 bd $\sqrt{f}$

The value of V_{cr} calculated at a particular section may be assumed to be constant for the distance equal to dl2, measured in the direction of increasing moment, from that particular section.

For a section cracked in flexure and with inclined tendons, the component of prestressing to the longitudinal axis of the members should be ignored.

Modes of failures due to shear:

For beams with low span-to depth ratio or inadequate shear reinforcement, the failure can be due to shear. A failure due to shear is sudden as compared to a failure due to flexure.

The following five modes of failure due to shear are identified,

1. Diagonal tension failure
2. Shear compression failure
3. Shear tension failure
4. Web crushing failure
5. Arch rib failure

The occurrence of the mode of failure depends on the span-to-depth ratio, loading, cross-section of the beam, amount and anchorage of reinforcement. The modes of failure are explained below:

1. **Diagonal tension failure**:

   In this mode, an inclined crack propagates rapidly due to inadequate shear reinforcement.

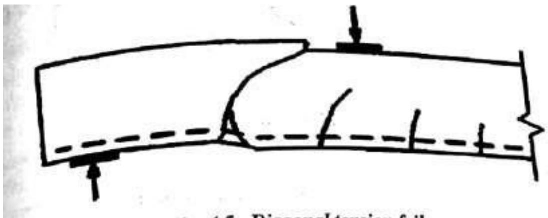


Fig. 4.5 : Diagonal tension failure

2. **Shear compression failure:**

   There is crushing of the concrete near the compression flange above the tip of the inclined crack.

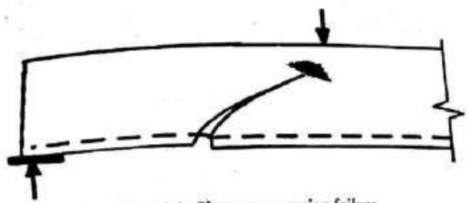
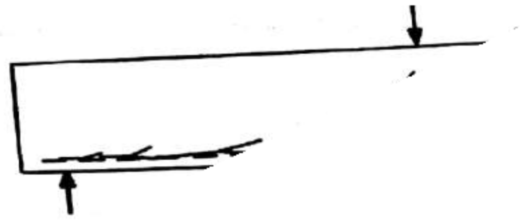


Fig. 4.6 : Shear compression failure

3. **Shear tension failure:**

   Due to inadequate anchorage of the longitudinal bars, the diagonal cracks propagate horizontally along the bars.



4. **<u>Web crushing failure:</u>**

   The concrete in the web crushes due to inadequate web thickness.

Fig. 4.8 : Web crushing failure

5. **<u>Arch rib failure:</u>**

For deep beams, the web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing.

Fig. 4.9 : Arch rib failure

The objective of design for shear is to avoid shear failure. The beam should fail in flexure its ultimate flexural strength. Hence, each mode of failure is addressed in the design for shear. The design involves not only the design of the stirrups, but also limiting the average shear stress in concrete, proving adequate thickness of the web and adequate development length of the longitudinal bars.

<u>Methods of improving the shear resistance of PSC members:</u>

There are three ways of improving the shear resistance of structural concrete members by prestressing techniques

1. Horizontal or axial prestressing
2. Prestressing by inclined or sloping cables and
3. Vertical or transverse pre-stressing

1. **Horizontal or axial prestressing**

Axial prestressing is defined as a member, in which the entire cross-section of concrete has a uniform compressive prestess. In this type of prestressing, the centroid of the tendons coincides with that of the concrete section.

2. **Prestressing by inclined or sloping cables**

In the design of reinforced concrete structures, tensile by bending generated by loading is resisted only by steel, delaying the cracking of the concrete. The reinforcement called passive reinforcement, intended to receive the tensile forces not absorbed by the concrete, working only when requested.

In prestressed concrete structures the force applied on the cables is transmitted to the concrete, resisting the tensile stresses by flexure and assisting the passive reinforcement. The prestressing reinforcement, called active reinforcement is placed in the structural element not only to compress and generate the necessary compressive stress so that the concrete can absorb the tensile stresses generated by loadings, but also resists to external loadings, providing many advantages, such as reduction of the shear forces by the action of the vertical component generated by the prestressing.

In the case of inclined cables , the increase of compressive stresses and reducing tensile stresses, vertical displacements and cracking reduction , with the structure remaining predominantly in the stage I along its useful life.

3. **Vertical or transverse pre-stressing**

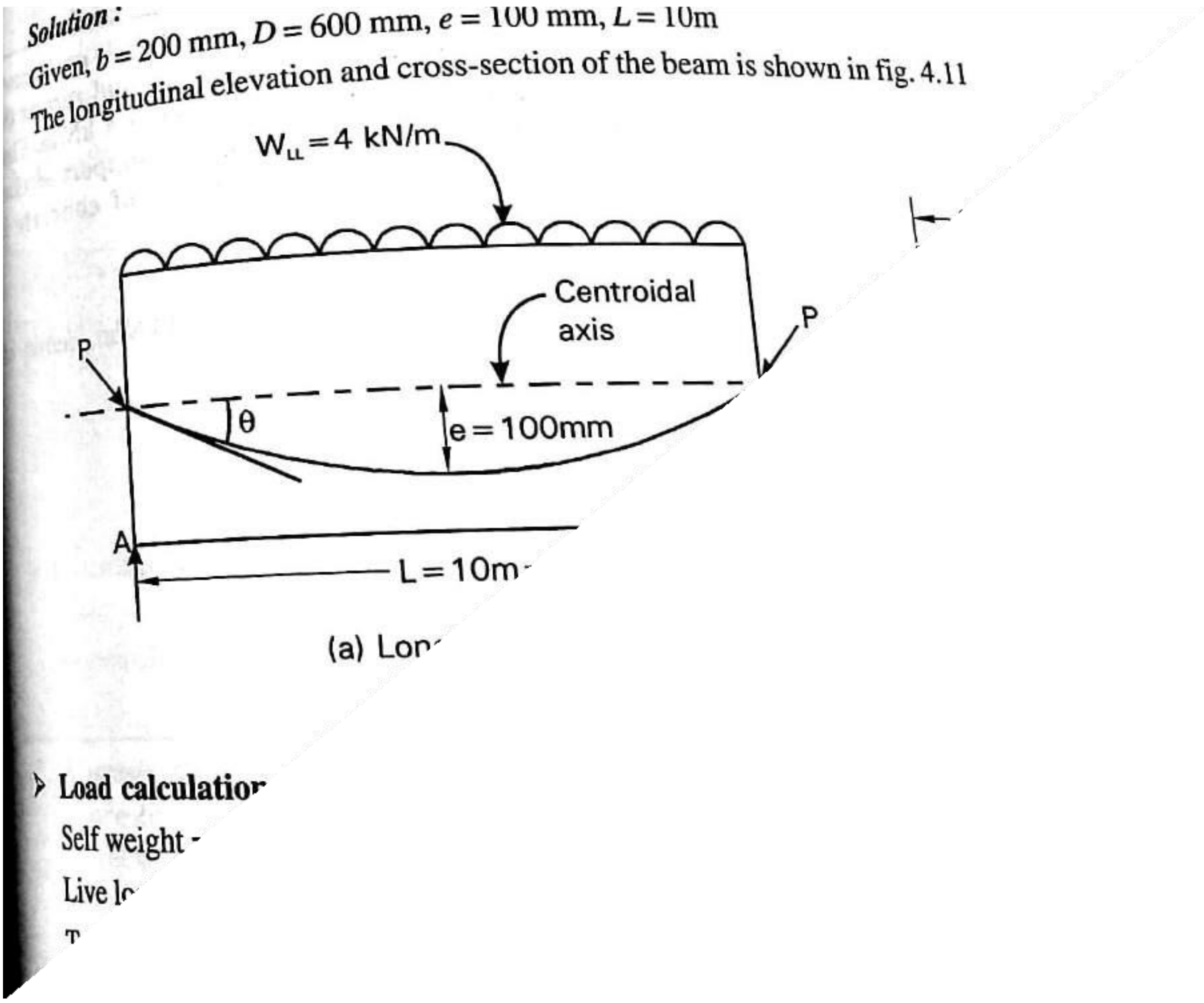
Prestressing concrete structures is generally performed to control flexural cracks because prestressing arranges the tendons in the axial direction of a given member.

On the other hand, in an attempt to delay the onset of shear cracking and to reduce the crack width, experimental studies have been conducted on reinforced concrete (RC) columns, which have been laterally prestressed by high – strength shear reinforcement.

The result of these flexure-shear tests have indicated that transverse prestressing increases the shear capacity at the first diagonal cracking (shear crack strength) and remarkably decreases the width of shear cracks, especially their residual openings.

This reduction of the crack opening improves earthquake resistance and durability.

Problems



Solution :

Given, $b = 200$ mm, $D = 600$ mm, $e = 100$ mm, $L = 10$m

The longitudinal elevation and cross-section of the beam is shown in fig. 4.11

➢ **Load calculatio**

Self weight

Live lo

➢ **Calculation of effective force**

$$V = \left(\frac{WL}{2}\right) = \left(\frac{6.88\times10}{}\right)$$

If, P = Prestressing force (effective force)

Slope of cable = sineϴ

$= \theta = \left(\frac{4e}{l}\right) = \left(\frac{4 \times 0.1}{}\right.$

Now, $V = P\sin\theta$

$$\Rightarrow \quad P\sin\theta = 34.40 \quad \Rightarrow P = \frac{34.40}{0.04}$$

$\therefore$ Effective force $= P = 860$ kN

- **Calculation of principal stresses**

Principal stresses or Direct stress $= \left(\frac{P}{A}\right) = \left(\frac{860 \times 10^3}{200 \times 600}\right) = 7.16$ N/mm²

A concrete beam of rectangular section, 200 mm wide and 400 ~
by a parabolic cable located at an eccentricity of 100 ~
supports. If the beam has a span of 10m and
the effective force necessary in the
For this condition
24 kN/m³

Solution :

Given, $b = 200$ mm, $D = 400$ mm, $e = 100$ mm or 0.1 m, $L = 10$m, $LL = 4$ kN/m, concrete = 24 kN/m³

- **Load calculation**

Self weight = (0.2 × 0.4 × 24) = 1.92 kN/m	
Live load (Given)	= 4 kN/m
Total load,	$W = 5.92$ kN/m

- **Calculation of effective force**

$$V = \frac{WL}{2} = \frac{5.92 \times 10}{2} = 29.6 \text{ kN}$$

$$\sin\theta = \frac{4e}{L} = \frac{4 \times 0.1}{10} = 0.04$$

$$P\sin\theta = V \Rightarrow P \times 0.04 = 29.6$$

$$\therefore P = \frac{29.6}{0.04} = 740 \text{ kN}$$

Principal stresses $= \frac{P}{A} = \frac{740 \times 10^3}{200 \times 400} = 9.25 \text{ N/mm}^2$

A concrete beam of rectangular section 200 mm wide and 650 mm deep is prestressed by a parabolic cable located at an eccentricity of 120 mm at midspan and zero at the supports. If the beam has a span of 12m and carries a uniformly distributed live load of 4.5 kN/m, find the effective force necessary in the cable for zero shear stress at the support section. For this condition, calculate the principal stresses. The density of concrete is 25 kN/m³.

Solution :

Given, $b = 200$ mm, $D = 650$ mm, $e = 120$ mm or 0.12m, $L = 12$m, $LL = 4.5$ kN/m, con

density = 25 kN/m³

- **Load calculation**

Self weight = 0.2 × 0.65 × 25 = 3.25 kN/m

Live load (Given) = 4.5 kN/m

Total load, $W = 7.75$ kN/m

- **Calculation of effective force**

$$V = \frac{WL}{2} = \frac{7.75 \times 12}{2}$$

$$\sin\theta = \frac{4e}{L} = \frac{4 \times}{}$$

$$P \sin\theta =$$

- **Calculation of**

Prin

5. A prestressed concrete beam of rectangular section, 300 mm wide by 600 mm deep is prestressed by two post-tensioned cables of area 600 mm² each initially stressed to 1600 N/mm². The cables are located at a constant eccentricity of 100 mm. The span of the beam is 10 m. If f_{ck} = 40 N/mm², estimate the ultimate shear resistance of support section uncracked flexure.

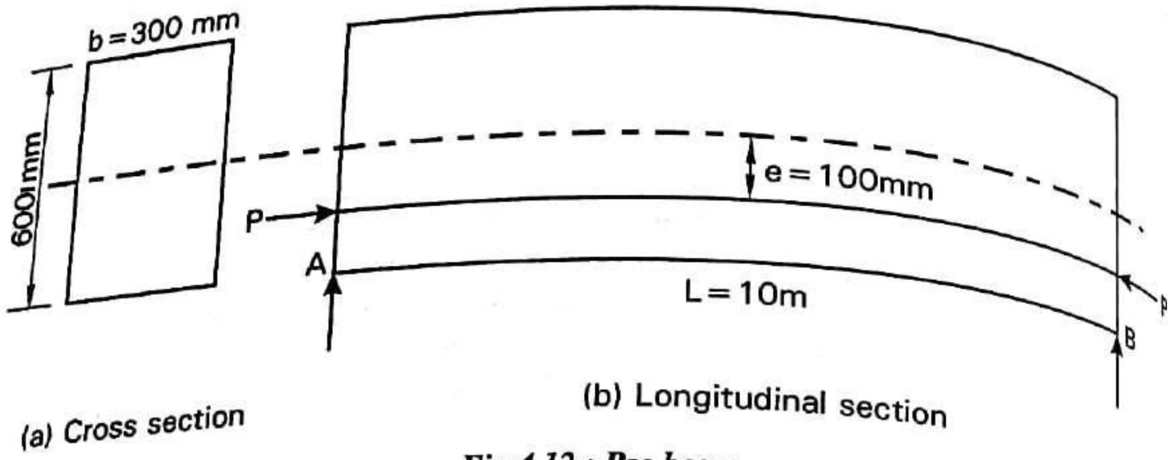


Fig 4.12 : Psc beam

Given data : $b = 300$ mm, $A_s = (2 \times 600) = 1200$ mm², $D = 600$ mm,
$A = (300 \times 600) = 18 \times 10^4$ mm², P = Area of post tensioned cable × initial stressess,
i.e., $P = (1200 \times 1600) = 192 \times 10^4$ N, $f_{ck} = 40$ N/mm², $e = 100$ mm.
Refer IS : 1343–1980, Clause 22.4.1.

- **Compressive prestress at centroid is given by the relation**

$$f_{cp} = \left(\frac{P}{A}\right) = \left(\frac{192 \times 10^4}{18 \times 10^4}\right) = 10.66 \text{ N/mm}^2$$

- **Tensile strength of concrete**

$$f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.517 \text{ N/mm}^2$$

- **Ultimate shear strength of support section uncracked in flexure** is given by the **relation**

$$V_{co} = 0.67bD\sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$= (0.67 \times 300 \times 600)\sqrt{1.517^2 + 0.8 \times 10.66 \times 1.517}$$

$$= 470776.8 \text{ N or } 470.77 \text{ kN}$$

The support section of a pre-stressed concrete beam 120 mm wide and 240 mm deep is required to support an ultimate shear force of 75 kN. The compressive pre stress at the centroidal axis is 5 MPa, f_{ck} = 40 MPa, f_y = 415 MPa. Concrete cover to shear reinforcemernt is 50 mm. Design a suitable shear reinforcement as per IS:1343

Solution :

Given, $b = 120$ mm, $D = 240$ mm, $V_u = 75$ kN, $f_{ck} = 40$ N/mm²

$f_r = f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.517$ N/mm², $f_y = 415$ N/mm², $f_{cp} = 5$ N/mm², $d' = 50$ mm,

$d_t = d = D - d' = 240 - 50 = 190$ mm

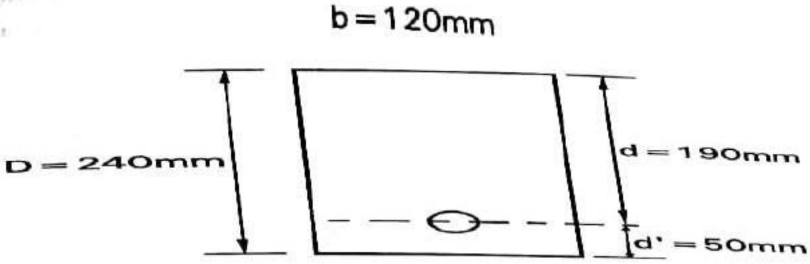


Fig. 4.13

➤ **For the support section uncracked in flexure, the shear strength is (Refer IS:1343-1980, Clause 22.4.1)**

$$V_{co} = 0.67\, bD\sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$= (0.67 \times 120 \times 240)\sqrt{1.517^2 + 0.8 \times 5 \times 1.517}$$

$$= 55822.79 \text{ N or } 55.82 \text{ kN}$$

Since $V_{co} < V_u$, shear reinforcement is to be provided.

➤ **Balance shear force,** $V_s = V_u - V_{co} = 75 - 55.82 = 19.18$ kN

Using 6mm diameter two legged stirrups

$$A_{sv} = \frac{\pi}{4} \times 6^2 = 56.54\ mm^2$$

$$\text{Spacing of stirrups} = S_v = \left[\frac{0.87 f_y d_t A_{sv}}{(V_u - V_{co})}\right]$$

$$= \left[\frac{0.87 \times 415 \times 190 \times 56.54}{19.18 \times 10^3}\right]$$

$$S_v = 202.22 \text{ mm}$$

Maximum spacing of stirrups not to exceed $0.75d$

$\therefore$ spacing $S_v \not> 0.75\,d = 0.75 \times 190 = 142.5$ mm

Hence, adopt 6mm diameter two legged stirrups at 140 mm centres.

If the support section of a PSC beam 100 mm wide and 250 mm deep is required support an ultimate shear force of 80 kN. The compressive prestress at the centroid axis is 5 N/mm². The characteristic cube strength of concrete is 40 N/mm². The cover to the tension reinforcement in 50mm. If the characteristic tensile strength stirrups is 415 N/mm², design suitable shear reinforcements in the section using code recommendations.

Solution :

Given, $b = 100$ mm, $D = 250$ mm, $V_u = 80$ kN, $f_{cp} = 5$ N/mm², $f_{ck} = 40$ N/mm², $f_y = 415$ N/mm², $d' = 50$ mm, $d = 250 - 50 = 200$mm, $f_t = f_v = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.517$ N/mm²

- **For the support section uncracked in flexure**

$$V_{co} = 0.67\, bD\sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$= 0.67 \times 100 \times 250 \times \sqrt{1.517^2 + 0.8 \times 5 \times 1.517} = 48457.2\text{ N}$$

$$= 48.457\text{ kN} < V_u\ (80\text{ kN})$$

$\therefore$ Shear reinforcement is to be provided

- **Balance shear force,** $V_s = V_u - V_{co} = 80 - 48.45 = 31.55$ kN

Using 6 mm diameter two legged stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2 = 56.54\text{ mm}^2$$

$$S_v = \left[\frac{0.87 f_y d A_{sv}}{V_s}\right] = \frac{0.87 \times 415 \times 200 \times 56.54}{(31.55 \times 10^3)}$$

$$= 129.40\text{ mm}$$

Maximum permisible spacing, $S_v \ngtr 0.75d$

$\therefore S_v = 0.75 \times 200 = 150$ mm

Hence, adopt 6mm diameter two legged stirrups at 120 mm centres.

The support section of a prestressed concrete beam 100 mm wide by 250 mm deep is required to support an ultimate shear force of 60 kN. The compressive prestress at centroid is 5 N/mm², f_{ck} = 40 N/mm², effective cover to reinforcement = 50mm. If f_y = 415 N/mm², design suitable shear reinforcement in the section using IS:1343 code recommedations.

Solution :

Given data $b = 100$ mm, $f_y = 415$ N/mm², $D = 250$ mm, $d' = 50$mm,

$\therefore d = D - d' = 250 - 50 = 200$ mm, $f_v = f_t = 0.24\sqrt{f_{ck}} = 0.24\sqrt{40} = 1.517$ N/mm²

$V_u = 60$ kN, $f_{ck} = 40$ N/mm², $f_{cp} = 5$ N/mm²

For the support section uncracked in flexure, the shear strength is

Refer IS:1343-1980, clause 22.4.1

$$V_{co} = 0.67\,bD\sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$= (0.67 \times 100 \times 250)\sqrt{1517^2 + (0.8 \times 5 \times 1517)} = 48457.28$$

$$= 48.45 \text{ kN} < V_u \text{ (60 kN)}$$

$\therefore$ Shear reinforcement is to be provided

Balance shear force, $V_s = (V_u - V_{co}) = (60 - 48.45) = 11.55$ kN

Using 6 mm diameter two legged stirrups, spacing is given by

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2 = 56.54 \text{ mm}^2$$

$$S_v = \left[\frac{A_{sv}.0.87 f_y d}{(V_u - V_{co})}\right]$$

$$= \left[\frac{56.54 \times 0.87 \times 415 \times 200}{11.55 \times 10^3}\right] = 353.48 \text{ mm}$$

Maximum permissible spacing $= S_v \not> 0.75d$

$$= (0.75 \times 200) = 150 \text{ mm}$$

Hence, adopt 6mm diameter two legged stirrups at 150 mm centres

The support section of a prestressed concrete beam, 120mm wide and 250 mm ddep, is required to support an ultimate shear force of 60 kN. The compressive prestress at the centroidal axis is 5 N/mm². The characteristic cube strength of concrete is 40 N/mm². The cover to the tension reinforcement is 50mm. If the characteristic tensile strength of steel in stirrups is 250 N/mm², design suitable reinforcements at the section using IS:1343 code specifications.

Solution :

Given data $V_u = 60$ kN, $b = 120$ mm, $D = 250$ mm, $f_v = f_y = 250$ N/mm²,

$f_{ck} = 40$ N/mm², $d' = 50$ mm, $f_t = 0.24\sqrt{40} = 1.517$ N/mm²

➢ **For the section uncracked in flexure, the shear strength is**

Refer IS:1343-1980, clause 22.4.1

$$V_{co} = 0.67\,bD\sqrt{f_t^2 + 0.8 f_{cp} f_t}$$

$$= (0.67 \times 120 \times 250)\sqrt{1517^2 + (0.8 \times 5 \times 1517)} = 58148.7\,N$$

$$= 58.14\text{ kN} < 60\text{ kN}$$

➢ **Balance shear force,** $V_s = (V_u - V_{co}) = (60 - 58) = 2$ kN

Using 6 mm diameter two legged stirrups,

$$A_{sv} = 2 \times \frac{\pi}{4} \times 6^2 = 56.54\text{ mm}^2$$

$$S_v = \left[\frac{A_{sv}.0.87 f_y d_t}{(V_u - V_{co})}\right]$$

$$= \left[\frac{56.54 \times 0.87 \times 250 \times 200}{2 \times 10^3}\right]$$

$$S_v = 1229.74\text{ mm}$$

Maximum spacing of stirrups not to exceed $0.75d$

$\therefore$ Spacing, $S_v \ngtr 0.75d = (0.75 \times 200) = 150$ mm

Hence, adopt 6mm diameter two legged stirrups at 150 mm centres.

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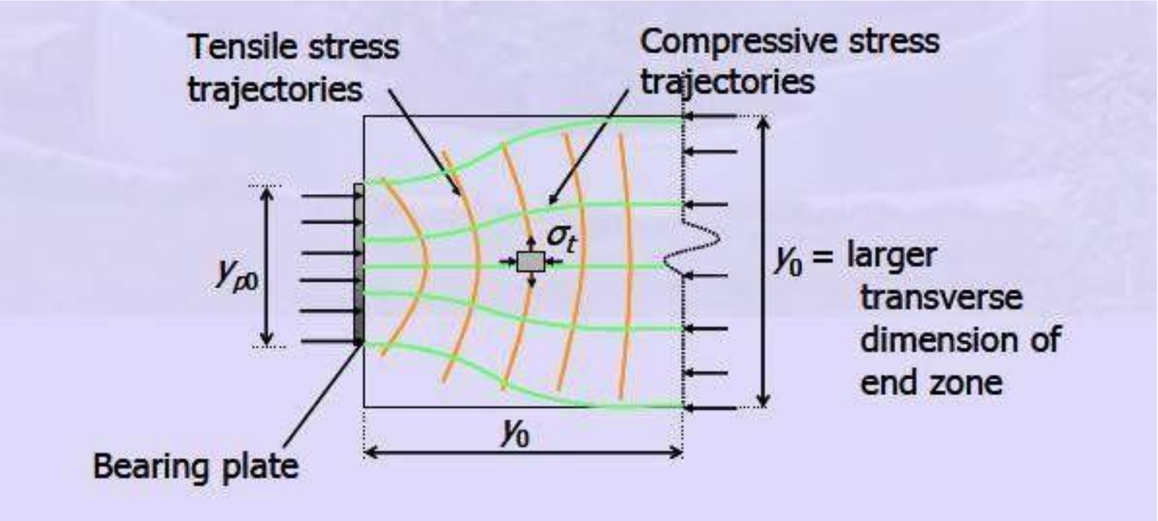
MODULE - 5

ANCHORAGE ZONE STRESSES AND COMPOSITE SECTIONS

End Block

Bursting force

A portion of a pre-stressed member surrounding the anchorage is the end block. Through the length of the end block, pre-stress is transferred from concentrated areas to become linearly distributed fiber stresses at the end of the block. The theoretical length of this block, called the lead length is not more than the height of the beam. But the stress distribution within this block is rather complicate.

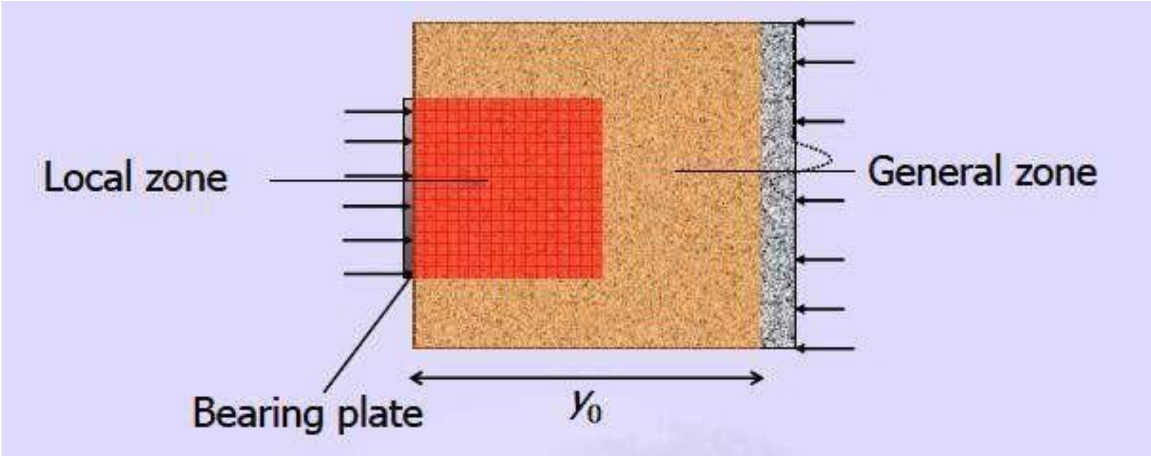


Stress trajectories in the end zone

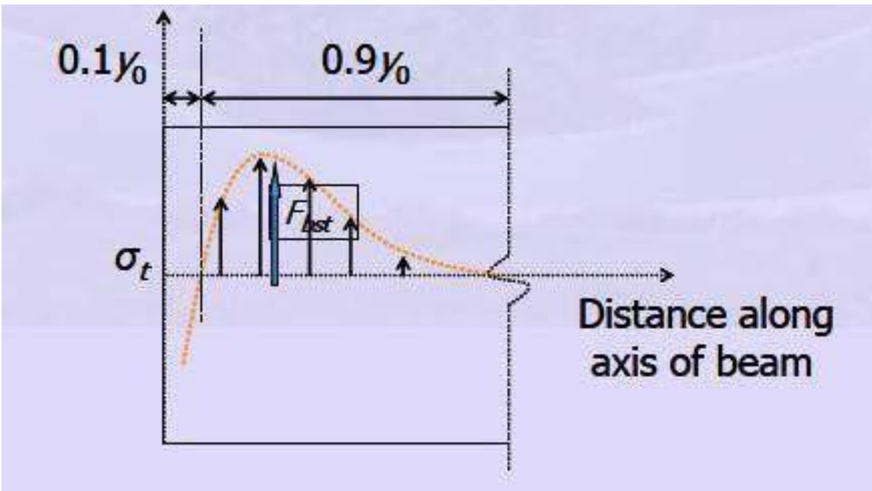
The larger transverse dimension of the end zone is represented as yo. The corresponding dimension of the bearing plate is represented as ypo. For analysis, the end zone is divided into a local zone and a general zone.

The local zone is the region behind the bearing plate and is subjected to high bearing stress and internal stresses. The behavior of the local zone is influenced by the anchorage device and the additional confining spiral reinforcement.

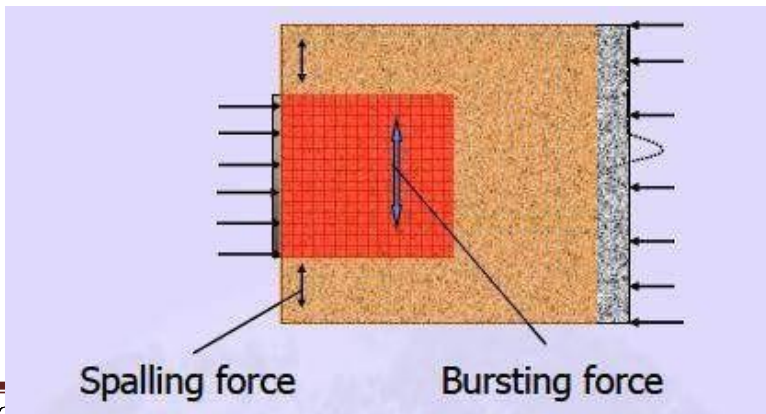
The general zone is the end zone region which is subjected to spalling of concrete. The zone is strengthened by end zone reinforcement.



The transverse stress (σ_t) at the CGC varies along the length of the end zone. It is compressive for a distance 0.1yo from the end and tensile thereafter, which drops down to zero at a distance yo from the end. The transverse tensile stress is known as splitting tensile stress. The resultant of the tensile stress in a transverse direction is known as the bursting force (F_{bst}).



Besides the bursting force there is spalling forces in the general zone.



F_{bst} for an individual square end zone loaded by a symmetrically placed square bearing plate according to Cl 18.6.2.2 is,

$$F_{bst} = P_K\left[0.32 - 0.3\frac{y_{po}}{y_o}\right]$$

Where, P_K = pre-stress in the tendon; y_{po} = length of a side of bearing plate; y_o = transverse dimension of end zone.

It can be observed that with the increase in size of the bearing plate the bursting force Fbst reduces.

End Zone Reinforcement

Transverse reinforcement - end zone reinforcement or anchorage zone reinforcement or bursting link - is provided in each principle direction based on the value of F_{bst}. The reinforcement is distributed within a length from 0.1yo to yo from an end of the member.

The amount of end zone reinforcement in each direction Ast is:

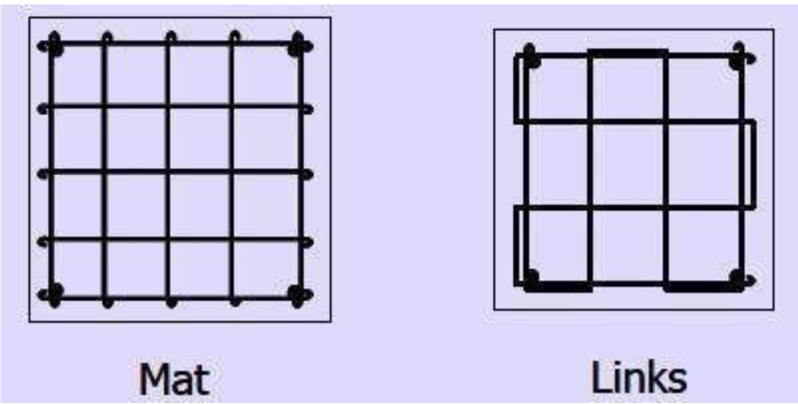
$$A_{st} = \frac{F_{bst}}{f_s}$$

The parameter represents the fraction of the transverse dimension covered by the bearing plate.

The stress in the transverse reinforcement, fs = 0.87fy.

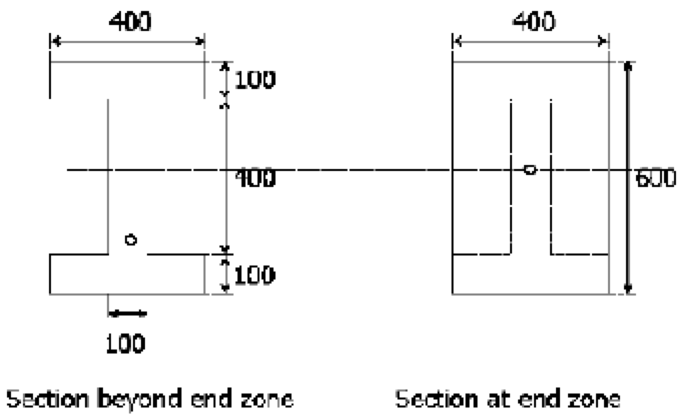
When the cover is less than 50 mm, fs = a value corresponding to a strain of 0.001.

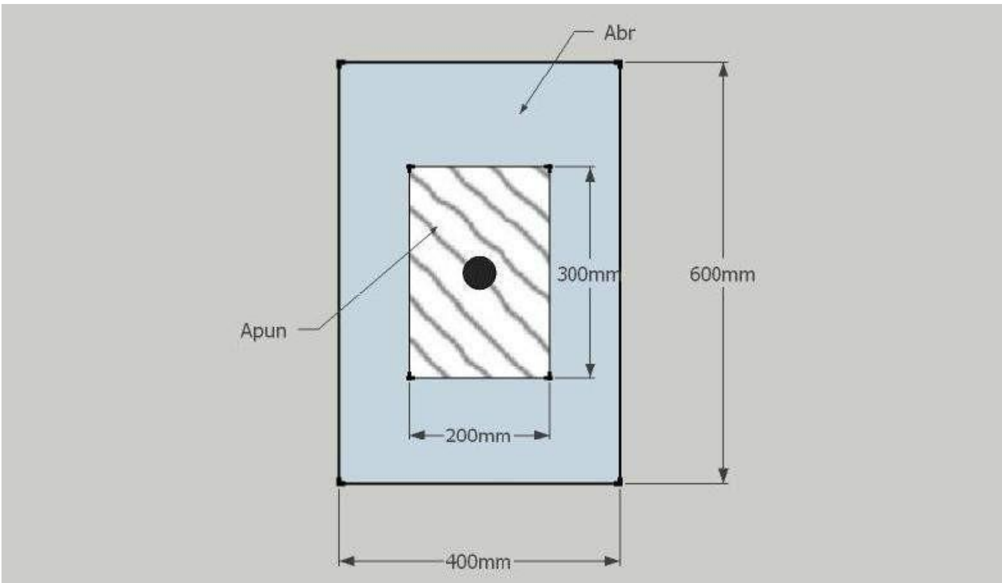
The end zone reinforcement is provided in several forms, some of which are proprietary of the construction firms. The forms are closed stirrups, mats or links with loops.



Bearing Plate & End Block

Design the bearing plate and the end zone reinforcement for the following bonded post-tensioned beam. The strength of concrete at transfer is 50 MPa. A pre-stressing force of 1055 kN is applied by a single tendon. There is no eccentricity of the tendon at the ends.





Bearing Plate

Assume area of bearing plate to be 200 mm x 300 mm

$$f_{br} = \frac{P_K}{A_{pun}}$$

P_K = 1055 kN

A_{pun} = 200 x 300 = 60000 mm^2

f_{br} = $\frac{1055\times10^3}{60000}$ = 17.58 MPa

A_{br} = 400 x 600 = 240000 mm^2

$$f_{br,all} = 0.48 f_{ci}\sqrt{\frac{A_{br}}{A_{pun}}}$$

= $0.48\times50\sqrt{\frac{240000}{60000}}$ = 48 MPa

$\leq 0.8\times f_{ci}$ = 40 MPa

$$f_{br} \leq f_{br,all} = 40 MPa$$

End Block

In vertical direction

$$F_{bst} = P_K\left[0.32 - 0.3\frac{y_{po}}{y_o}\right]$$

= $1055\left[0.32 - 0.3\ \frac{300}{600}\right]$ = 179.35 kN

In horizontal direction

$$F_{bst} = P_K\left[0.32 - 0.3\frac{y_{po}}{y_o}\right]$$

= $1055\left[0.32 - 0.3\ \frac{200}{400}\right]$ = 179.35 kN

$A_{st} = \dfrac{F_{bst}}{0.87 f_y}$

$= \dfrac{179.35 \times 10^3}{0.87 \times 250}$ $= 824.60\ mm^2$

Provide 10 mm 2L stirrups in both directions as F_{bst} is same in those

$A_w = \dfrac{\pi \times 10^2}{4}$ $= 78.54\ mm^2$

No of stirrups $= \dfrac{824.60}{2 \times 78.54}$ = 6 Nos

Provide $\dfrac{2}{3}^{rd}$ A_{st} from 0.1 y_o = 60 mm to 0.5 y_o = 300 mm and $\dfrac{1}{3}^{rd}$ A_{st} from 0.5 y_o = 300 mm to y_o = 600 mm, both vertically and horizontal.

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